Package 'genridge'

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Type Package Title Generalized Ridge Trace Plots for Ridge Regression Version 0.8.0 Date 2024-11-30 Maintainer Michael Friendly <friendly@yorku.ca> **Depends** R $(>= 3.5.0)$, car Imports rgl, colorspace, splines Suggests MASS, bestglm, vcdExtra Description The genridge package introduces generalizations of the standard univariate ridge trace plot used in ridge regression and related methods. These graphical methods show both bias (actually, shrinkage) and precision, by plotting the covariance ellipsoids of the estimated coefficients, rather than just the estimates themselves. 2D and 3D plotting methods are provided, both in the space of the predictor variables and in the transformed space of the PCA/SVD of the predictors. License GPL $(>= 2)$ LazyLoad yes LazyData yes Language en-US BugReports <https://github.com/friendly/genridge/issues> URL <https://github.com/friendly/genridge>, <https://friendly.github.io/genridge/> RoxygenNote 7.3.2 Encoding UTF-8 NeedsCompilation no Author Michael Friendly [aut, cre] (<<https://orcid.org/0000-0002-3237-0941>>) Repository CRAN Date/Publication 2024-12-02 15:00:02 UTC

Contents

genridge-package *Generalized ridge trace plots for ridge regression*

Description

The genridge package introduces generalizations of the standard univariate ridge trace plot used in ridge regression and related methods (Friendly, 2012). These graphical methods show both bias (actually, shrinkage) and precision, by plotting the covariance ellipsoids of the estimated coefficients, rather than just the estimates themselves. 2D and 3D plotting methods are provided, both in the space of the predictor variables and in the transformed space of the PCA/SVD of the predictors.

Details

This package provides computational support for the graphical methods described in Friendly (2013). Ridge regression models may be fit using the function [ridge](#page-26-1), which incorporates features of [lm.ridge](#page-0-0). In particular, the shrinkage factors in ridge regression may be specified either in terms of the constant added to the diagonal of X^TX matrix (lambda), or the equivalent number of degrees of freedom.

More importantly, the [ridge](#page-26-1) function also calculates and returns the associated covariance matrices of each of the ridge estimates, allowing precision to be studied and displayed graphically.

This provides the support for the main plotting functions in the package:

[plot.ridge](#page-17-1): Bivariate ridge trace plots

[pairs.ridge](#page-12-1): All pairwise bivariate ridge trace plots

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[plot3d.ridge](#page-20-1): 3D ridge trace plots

[traceplot](#page-30-1): Traditional univariate ridge trace plots

In addition, the function [pca.ridge](#page-14-1) transforms the coefficients and covariance matrices of a ridge object from predictor space to the equivalent, but more interesting space of the PCA of $X^T X$ or the SVD of X. The main plotting functions also work for these objects, of class c("ridge", "pcaridge").

Finally, the functions [precision](#page-23-1) and [vif.ridge](#page-33-1) provide other useful measures and plots.

Author(s)

Michael Friendly

Maintainer: Michael Friendly <friendly@yorku.ca>

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, 22(1), 50-68, doi:10.1080/10618600.2012.681237, <https://www.datavis.ca/papers/genridge-jcgs.pdf>

Arthur E. Hoerl and Robert W. Kennard (1970). Ridge Regression: Biased Estimation for Nonorthogonal Problems, *Technometrics*, 12(1), pp. 55-67.

Arthur E. Hoerl and Robert W. Kennard (1970). Ridge Regression: Applications to Nonorthogonal Problems *Technometrics*, 12(1), pp. 69-82.

See Also

[lm.ridge](#page-0-0)

Examples

see examples for ridge, etc.

Acetylene *Acetylene Data*

Description

The data consist of measures of yield of a chemical manufacturing process for acetylene in relation to numeric parameters.

Format

A data frame with 16 observations on the following 4 variables.

yield conversion percentage yield of acetylene

temp reactor temperature (celsius)

ratio H2 to N-heptone ratio

time contact time (sec)

Details

Marquardt and Snee (1975) used these data to illustrate ridge regression in a model containing quadratic and interaction terms, particularly the need to center and standardize variables appearing in high-order terms.

Typical models for these data include the interaction of temp:ratio, and a squared term in temp

Source

SAS documentation example for PROC REG, *Ridge Regression for Acetylene Data*.

References

Marquardt, D.W., and Snee, R.D. (1975), "Ridge Regression in Practice," *The American Statistician*, 29, 3-20.

Marquardt, D.W. (1980), "A Critique of Some Ridge Regression Methods: Comment," *Journal of the American Statistical Association*, Vol. 75, No. 369 (Mar., 1980), pp. 87-91

Examples

```
data(Acetylene)
```

```
# naive model, not using centering
amod0 <- lm(yield \sim temp + ratio + time + I(time^2) + temp:time, data=Acetylene)y <- Acetylene[,"yield"]
X0 <- model.matrix(amod0)[,-1]
lambda <- c(0, 0.0005, 0.001, 0.002, 0.005, 0.01)
aridge0 <- ridge(y, X0, lambda=lambda)
traceplot(aridge0)
traceplot(aridge0, X="df")
pairs(aridge0, radius=0.2)
```
biplot.pcaridge *Biplot of Ridge Regression Trace Plot in SVD Space*

Description

biplot.pcaridge supplements the standard display of the covariance ellipsoids for a ridge regression problem in PCA/SVD space with labeled arrows showing the contributions of the original variables to the dimensions plotted.

biplot.pcaridge 5

Usage

```
## S3 method for class 'pcaridge'
biplot(
 x,
 variables = (p - 1):p,
 labels = NULL,
 asp = 1,origin,
 scale,
 var.lab = rownames(V),
 var.lwd = 1,
 var.col = "black",
 var.cex = 1,
 xlab,
 ylab,
 prefix = "Dim ",
 suffix = TRUE,...
```
)

Arguments

... Other arguments, passed to [plot.pcaridge](#page-17-2)

Details

The biplot view showing the dimensions corresponding to the two *smallest* singular values is particularly useful for understanding how the predictors contribute to shrinkage in ridge regression.

This is only a biplot in the loose sense that results are shown in two spaces simultaneously – the transformed PCA/SVD space of the original predictors, and vectors representing the predictors projected into this space.

biplot.ridge is a similar extension of [plot.ridge](#page-17-1), adding vectors showing the relation of the PCA/SVD dimensions to the plotted variables.

class("ridge") objects use the transpose of the right singular vectors, $t(x$ \$svd. $V)$ for the dimension weights plotted as vectors.

Value

None

Author(s)

Michael Friendly, with contributions by Uwe Ligges

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, 22(1), 50-68, [doi:10.1080/10618600.2012.681237,](https://doi.org/10.1080/10618600.2012.681237) <https://datavis.ca/papers/genridge-jcgs.pdf>

See Also

[plot.ridge](#page-17-1), [pca.ridge](#page-14-1)

Examples

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])
lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)
plridge <- pca(lridge)
plot(plridge, radius=0.5)
# same, with variable vectors
biplot(plridge, radius=0.5)
# add some other options
biplot(plridge, radius=0.5, var.col="brown", var.lwd=2, var.cex=1.2, prefix="Dimension ")
```
biplots for ridge objects, showing PCA vectors

```
plot(lridge, radius=0.5)
biplot(lridge, radius=0.5)
biplot(lridge, radius=0.5, asp=NA)
```
contourf *Enhanced Contour Plots*

Description

This is an enhancement to [contour](#page-0-0), written as a wrapper for that function. It creates a contour plot, or adds contour lines to an existing plot, allowing the contours to be filled and returning the list of contour lines.

Usage

```
contourf(
  x = seq(0, 1, length.out = nrow(z)),y = seq(0, 1, length.out = ncol(z)),z,
 nlevels = 10,
  levels = pretty(zlim, nlevels),
  zlim = range(z, finite = TRUE),col = par("fg").color.palette = colorRampPalette(c("white", col)),
  fill.col = color.palette(nlevels + 1),
  fill.alpha = 0.5,
  add = FALSE,...
\mathcal{L}
```
Arguments

Value

Returns invisibly the list of contours lines, with components levels, x, y. See [contourLines](#page-0-0).

Author(s)

Michael Friendly

See Also

[contour](#page-0-0), [contourLines](#page-0-0)

[contourplot](#page-0-0) from package lattice.

Examples

```
x <- 10*1:nrow(volcano)
y <- 10*1:ncol(volcano)
contourf(x,y,volcano, col="blue")
contourf(x,y,volcano, col="blue", nlevels=6)
# return value, unfilled, other graphic parameters
```

```
res <- contourf(x,y,volcano, col="blue", fill.col=NULL, lwd=2)
# levels used in the plot
sapply(res, function(x) x[[1]])
```
Detroit *Detroit Homicide Data for 1961-1973*

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Description

The data set Detroit was used extensively in the book by Miller (2002) on subset regression. The data are unusual in that a subset of three predictors can be found which gives a very much better fit to the data than the subsets found from the Efroymson stepwise algorithm, or from forward selection or backward elimination. They are also unusual in that, as time series data, the assumption of independence is patently violated, and the data suffer from problems of high collinearity.

As well, ridge regression reveals somewhat paradoxical paths of shrinkage in univariate ridge trace plots, that are more comprehensible in multivariate views.

Format

A data frame with 13 observations on the following 14 variables.

Police Full-time police per 100,000 population

Unemp Percent unemployed in the population

MfgWrk Number of manufacturing workers in thousands

GunLic Number of handgun licences per 100,000 population

GunReg Number of handgun registrations per 100,000 population

HClear Percent of homicides cleared by arrests

WhMale Number of white males in the population

NmfgWrk Number of non-manufacturing workers in thousands

GovWrk Number of government workers in thousands

HrEarn Average hourly earnings

WkEarn Average weekly earnings

Accident Death rate in accidents per 100,000 population

Assaults Number of assaults per 100,000 population

Homicide Number of homicides per 100,000 of population

Details

The data were originally collected and discussed by Fisher (1976) but the complete dataset first appeared in Gunst and Mason (1980, Appendix A). Miller (2002) discusses this dataset throughout his book, but doesn't state clearly which variables he used as predictors and which is the dependent variable. (Homicide was the dependent variable, and the predictors were Police . . . WkEarn.) The data were obtained from StatLib.

A similar version of this data set, with different variable names appears in the bestglm package.

Source

<https://lib.stat.cmu.edu/datasets/detroit>

References

Fisher, J.C. (1976). Homicide in Detroit: The Role of Firearms. *Criminology*, 14, 387–400.

Gunst, R.F. and Mason, R.L. (1980). *Regression analysis and its application: A data-oriented approach*. Marcel Dekker.

Miller, A. J. (2002). *Subset Selection in Regression*. 2nd Ed. Chapman & Hall/CRC. Boca Raton.

Examples

```
data(Detroit)
```

```
# Work with a subset of predictors, from Miller (2002, Table 3.14),
# the "best" 6 variable model
# Variables: Police, Unemp, GunLic, HClear, WhMale, WkEarn
# Scale these for comparison with other methods
Det \leq as.data.frame(scale(Detroit[,c(1,2,4,6,7,11)]))
Det <- cbind(Det, Homicide=Detroit[,"Homicide"])
# use the formula interface; specify ridge constants in terms
# of equivalent degrees of freedom
dridge <- ridge(Homicide ~ ., data=Det, df=seq(6,4,-.5))
# univariate trace plots are seemingly paradoxical in that
# some coefficients "shrink" *away* from 0
traceplot(dridge, X="df")
vif(dridge)
pairs(dridge, radius=0.5)
plot3d(dridge, radius=0.5, labels=dridge$df)
# transform to PCA/SVD space
dpridge <- pca(dridge)
# not so paradoxical in PCA space
traceplot(dpridge, X="df")
biplot(dpridge, radius=0.5, labels=dpridge$df)
```

```
# show PCA vectors in variable space
biplot(dridge, radius=0.5, labels=dridge$df)
```
diab 11 and 12 and 12 and 13 and 13 and 13 and 13 and 13 and 14 and 15 and 16 and 17 and 17 and 17 and 17 and 1

Description

These data consist of observations on 442 patients, with the response of interest being a quantitative measure of disease progression one year after baseline.

There are ten baseline variables: age, sex, body-mass index (bmi), average blood pressure (map) and six blood serum measurements.

Usage

data("diab")

Format

A data frame with 442 observations on the following 11 variables.

prog disease progression, a numeric vector

- age age, a numeric vector
- sex integer, a numeric vector
- bmi body mass index, a numeric vector
- map mean arterial blood pressure, a numeric vector
- tc blood serum TC, a numeric vector
- ldl blood serum low-density lipoprotein ("bad cholersterol"), a numeric vector
- hdl blood serum high-density lipoprotein ("good cholersterol"), a numeric vector
- tch blood serum TCH, a numeric vector
- ltg blood serum lamotrigine, a numeric vector
- glu blood serum glucose, a numeric vector

Details

Efron & Hastie describe their analysis using the centered predictor variables standardized to unit L2 norm. [ridge](#page-26-1) does not (yet) provide this scaling.

Source

The dataset was taken from the web site for Efron & Hastie (2021), *Computer Age Statistical Inference*, https://hastie.su.domains/CASI_files/DATA/diabetes.csv.

References

Efron, B., Hastie, T., Johnstone, I., & Tibshirani, R. (2004). Least Angle Regression. *The Annals of Statistics*, 32(2), 407-499. [doi:10.1214/009053604000000067](https://doi.org/10.1214/009053604000000067)

Efron, B., & Hastie, T. (2021). *Computer Age Statistical Inference, Student Edition: Algorithms, Evidence, and Data Science*, Cambridge University Press. [doi:10.1017/9781108914062](https://doi.org/10.1017/9781108914062)

Examples

data(diab) ## maybe str(diab) ; plot(diab) ...

Description

The hospital manpower data, taken from Myers (1990), table 3.8, are a well-known example of highly collinear data to which ridge regression and various shrinkage and selection methods are often applied.

The data consist of measures taken at 17 U.S. Naval Hospitals and the goal is to predict the required monthly man hours for staffing purposes.

Format

A data frame with 17 observations on the following 6 variables.

Hours monthly man hours (response variable)

Load average daily patient load

Xray monthly X-ray exposures

BedDays monthly occupied bed days

AreaPop eligible population in the area in thousands

Stay average length of patient's stay in days

Details

Myers (1990) indicates his source was "Procedures and Analysis for Staffing Standards Development: Data/Regression Analysis Handbook", Navy Manpower and Material Analysis Center, San Diego, 1979.

Source

Raymond H. Myers (1990). *Classical and Modern Regression with Applications*, 2nd ed., PWS-Kent, pp. 130-133.

References

Donald R. Jensen and Donald E. Ramirez (2012). Variations on Ridge Traces in Regression, *Communications in Statistics - Simulation and Computation*, 41 (2), 265-278.

See Also

[manpower](#page-0-0) for the same data, and other analyses

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Examples

```
data(Manpower)
mmod <- lm(Hours ~ ., data=Manpower)
vif(mmod)
# ridge regression models, specified in terms of equivalent df
mridge \le ridge(Hours \sim ., data=Manpower, df=seq(5, 3.75, -.25))
vif(mridge)
# univariate ridge trace plots
traceplot(mridge)
traceplot(mridge, X="df")
# bivariate ridge trace plots
plot(mridge, radius=0.25, labels=mridge$df)
pairs(mridge, radius=0.25)
# 3D views
# ellipsoids for Load, Xray & BedDays are nearly 2D
plot3d(mridge, radius=0.2, labels=mridge$df)
# variables in model selected by AIC & BIC
plot3d(mridge, variables=c(2,3,5), radius=0.2, labels=mridge$df)
# plots in PCA/SVD space
mpridge <- pca(mridge)
traceplot(mpridge, X="df")
```
biplot(mpridge, radius=0.25)

pairs.ridge *Scatterplot Matrix of Bivariate Ridge Trace Plots*

Description

Displays all possible pairs of bivariate ridge trace plots for a given set of predictors.

Usage

```
## S3 method for class 'ridge'
pairs(
  x,
  variables,
  radius = 1,
  1wd = 1,
  lty = 1,col = c("black", "red", "darkgreen", "blue", "darkcyan", "magenta", "brown",
    "darkgray"),
```

```
center.pch = 16,
 center.cex = 1.25,
 digits = getOption("digits") - 3,
 diag.cex = 2,
 diag.panel = panel.label,
 fill = FALSE,fill.alpha = 0.3,
  ...
)
```
Arguments

Value

None. Used for its side effect of plotting.

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, 22(1), 50-68, doi:10.1080/10618600.2012.681237, <https://www.datavis.ca/papers/genridge-jcgs.pdf>

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See Also

[ridge](#page-26-1) for details on ridge regression as implemented here

[plot.ridge](#page-17-1), [traceplot](#page-30-1) for other plotting methods

Examples

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])
lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)
pairs(lridge, radius=0.5, diag.cex=1.75)
data(prostate)
py <- prostate[, "lpsa"]
pX <- data.matrix(prostate[, 1:8])
pridge <- ridge(py, pX, df=8:1)
pairs(pridge)
```
pca *Transform Ridge Estimates to PCA Space*

Description

The function pca.ridge transforms a ridge object from parameter space, where the estimated coefficients are β_k with covariance matrices Σ_k , to the principal component space defined by the right singular vectors, V , of the singular value decomposition of the scaled predictor matrix, X .

In this space, the transformed coefficients are $V\beta_k$, with covariance matrices

 $V\Sigma_k V^T$

This transformation provides alternative views of ridge estimates in low-rank approximations. In particular, it allows one to see where the effects of collinearity typically reside — in the smallest PCA dimensions.

Usage

.

 $pca(x, \ldots)$

Arguments

Value

An object of class c("ridge", "pcaridge"), with the same components as the original ridge object.

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, 22(1), 50-68, doi:10.1080/10618600.2012.681237, <https://www.datavis.ca/papers/genridge-jcgs.pdf>

See Also

[ridge](#page-26-1)

Examples

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])
lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)
plridge <- pca(lridge)
traceplot(plridge)
pairs(plridge)
```
view in space of smallest singular values

plot(plridge, variables=5:6)

plot.precision *Plot Shrinkage vs. Variance for Ridge Precision*

Description

This function uses the results of [precision](#page-23-1) to plot a measure of shrinkage of the coefficients in ridge regression against a selected measure of their estimated sampling variance, so as to provide a direct visualization of the tradeoff between bias and precision.

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Usage

```
## S3 method for class 'precision'
plot(
 x,
 xvar = "norm.beta",
 yvar = c("det", "trace", "max.eig"),
 labels = c("lambda", "df"),
 label.cex = 1.25,label.prefix,
 criteria = NULL,
 pch = 16,
 cex = 1.5,
 col,
 main = NULL,
 xlab,
 ylab,
  ...
\mathcal{L}
```
Arguments

Value

Returns nothing. Used for the side effect of plotting.

Author(s)

Michael Friendly

See Also

[ridge](#page-26-1) for details on ridge regression as implemented here. [precision](#page-23-1) for definitions of the measures

Examples

```
lambda <- c(0, 0.001, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(Employed ~ GNP + Unemployed + Armed.Forces +
                  Population + Year + GNP.deflator,
                data=longley, lambda=lambda)
criteria <- lridge$criteria |> print()
pridge <- precision(lridge) |> print()
plot(pridge)
# also show optimal criteria
plot(pridge, criteria = criteria)
# use degrees of freedom as point labels
plot(pridge, labels = "df")
plot(pridge, labels = "df", label.prefix="df:")
# show the trace measure
plot(pridge, yvar="trace")
```
plot.ridge *Bivariate Ridge Trace Plots*

Description

The bivariate ridge trace plot displays 2D projections of the covariance ellipsoids for a set of ridge regression estimates indexed by a ridge tuning constant.

The centers of these ellipses show the bias induced for each parameter, and also how the change in the ridge estimate for one parameter is related to changes for other parameters.

The size and shapes of the covariance ellipses show directly the effect on precision of the estimates as a function of the ridge tuning constant.

plot.pcaridge does these bivariate ridge trace plots for "pcaridge" objects, defaulting to plotting the two smallest components.

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Usage

```
## S3 method for class 'ridge'
plot(
  x,
  variables = 1:2,
  radius = 1,which.lambda = 1:length(x$lambda),
  labels = lambda,
  pos = 3,
  cex = 1.2,
  1wd = 2,\frac{1}{y} = 1,xlim,
  ylim,
  col = c("black", "red", "darkgreen", "blue", "darkcyan", "magenta", "brown",
    "darkgray"),
  center.pch = 16,
  center.cex = 1.5,
  fill = FALSE,fill.alpha = 0.3,
  ref = TRUE,ref.col = gray(0.7),
  ...
\mathcal{L}
```
S3 method for class 'pcaridge' $plot(x, variables = (p - 1):p, labels = NULL, ...)$

Arguments

Value

None. Used for its side effect of plotting.

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, 22(1), 50-68, doi:10.1080/10618600.2012.681237, <https://www.datavis.ca/papers/genridge-jcgs.pdf>

See Also

[ridge](#page-26-1) for details on ridge regression as implemented here; [pairs.ridge](#page-12-1), [traceplot](#page-30-1), for basic plots.

[pca.ridge](#page-14-1) for transformation of ridge regression estimates to PCA space. [biplot.pcaridge](#page-3-1) and [plot3d.ridge](#page-20-1) for other plotting methods

Examples

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])
```

```
lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lambdaf <- c("", ".005", ".01", ".02", ".04", ".08")
lridge <- ridge(longley.y, longley.X, lambda=lambda)
```

```
op <- par(mfrow=c(2,2), mar=c(4, 4, 1, 1)+ 0.1)
for (i in 2:5) {
plot(lridge, variables=c(1,i), radius=0.5, cex.lab=1.5)
text(lridge$coef[1,1], lridge$coef[1,i], expression(~widehat(beta)^OLS),
```
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```
cex=1.5, pos=4, offset=.1)
if (i==2) text(lridge$coef[-1,1:2], lambdaf[-1], pos=3, cex=1.25)
}
par(op)
data(prostate)
py <- prostate[, "lpsa"]
pX <- data.matrix(prostate[, 1:8])
pridge <- ridge(py, pX, df=8:1)
plot(pridge)
plot(pridge, fill=c(TRUE, rep(FALSE,7)))
```
plot3d *3D Ridge Trace Plots*

Description

The 3D ridge trace plot displays 3D projections of the covariance ellipsoids for a set of ridge regression estimates indexed by a ridge tuning constant.

The centers of these ellipses show the bias induced for each parameter, and also how the change in the ridge estimate for one parameter is related to changes for other parameters.

The size and shapes of the covariance ellipsoids show directly the effect on precision of the estimates as a function of the ridge tuning constant.

plot3d.ridge and plot3d.pcaridge differ only in the defaults for the variables plotted.

Usage

```
plot3d(x, \ldots)## S3 method for class 'pcaridge'
plot3d(x, variables = (p - 2):p, ...)## S3 method for class 'ridge'
plot3d(
  x,
  variables = 1:3,
  radius = 1,
  which.lambda = 1:length(x$lambda),
  1wd = 1,
  lty = 1,
  xlim,
  ylim,
  zlim,
  xlab,
```

```
ylab,
zlab,
col = c("black", "red", "darkgreen", "blue", "darkcyan", "magenta", "brown",
  "darkgray"),
labels = lambda,
ref = TRUE,ref.col = gray(0.7),
segments = 40,
shade = TRUE,
shade.alpha = 0.1,
wire = FALSE,
aspect = 1,
add = FALSE,
...
```
Arguments

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Value

None. Used for its side-effect of plotting

Note

This is an initial implementation. The details and arguments are subject to change.

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, 22(1), 50-68, doi:10.1080/10618600.2012.681237, <https://www.datavis.ca/papers/genridge-jcgs.pdf>

See Also

[plot.ridge](#page-17-1), [pairs.ridge](#page-12-1), [pca.ridge](#page-14-1)

Examples

```
lmod <- lm(Employed ~ GNP + Unemployed + Armed.Forces + Population +
                     Year + GNP.deflator, data=longley)
longley.y <- longley[, "Employed"]
longley.X <- model.matrix(lmod)[,-1]
```

```
lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lambdaf <- c("0", ".005", ".01", ".02", ".04", ".08")
lridge <- ridge(longley.y, longley.X, lambda=lambda)
```

```
plot3d(lridge, var=c(1,4,5), radius=0.5)
```

```
# view in SVD/PCA space
plridge <- pca(lridge)
```
plot3d(plridge, radius=0.5)

precision *Measures of Precision and Shrinkage for Ridge Regression*

Description

The goal of precision is to allow you to study the relationship between shrinkage of ridge regression coefficients and their precision directly by calculating measures of each.

Three measures of (inverse) precision based on the "size" of the covariance matrix of the parameters are calculated. Let $V_k \equiv \text{Var}(\beta_k)$ be the covariance matrix for a given ridge constant, and let λ_i , $i = 1, \dots p$ be its eigenvalues. Then the variance (= 1/precision) measures are:

- 1. "det": $\log |V_k| = \log \prod \lambda$ (with det. fun = "log", the default) or $|V_k|^{1/p} = (\prod \lambda)^{1/p}$ (with det.fun = "root") measures the linearized volume of the covariance ellipsoid and corresponds conceptually to Wilks' Lambda criterion
- 2. "trace": trace(V_k) = $\sum \lambda$ corresponds conceptually to Pillai's trace criterion
- 3. "max.eig": $\lambda_1 = \max(\lambda)$ corresponds to Roy's largest root criterion.

Two measures of shrinkage are also calculated:

- norm.beta: the root mean square of the coefficient vector $\|\beta_k\|$, normalized to a maximum of 1.0 if normalize == TRUE (the default).
- norm.diff: the root mean square of the difference from the OLS estimate $||\beta_{OLS} \beta_k||$. This measure is inversely related to norm.beta

A plot method, [plot.precision](#page-15-1) facilitates making graphs of these quantities.

Usage

```
precision(object, det.fun, normalize, ...)
```
Arguments

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Value

An object of class c("precision", "data.frame") with the following columns:

Note

Models fit by lm and ridge use a different scaling for the predictors, so the results of precision for an Im model will not correspond to those for ridge with ridge constant = 0.

Author(s)

Michael Friendly

See Also

[ridge](#page-26-1), [plot.precision](#page-15-1)

Examples

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])
lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)
# same, using formula interface
lridge <- ridge(Employed ~ GNP + Unemployed + Armed.Forces + Population + Year + GNP.deflator,
data=longley, lambda=lambda)
clr <- c("black", rainbow(length(lambda)-1, start=.6, end=.1))
coef(lridge)
(pdat <- precision(lridge))
# plot log |Var(b)| vs. length(beta)
with(pdat, {
plot(norm.beta, det, type="b",
cex.lab=1.25, pch=16, cex=1.5, col=clr, lwd=2,
xlab='shrinkage: ||b|| / max(||b||)',
ylab='variance: log |Var(b)|')
text(norm.beta, det, lambda, cex=1.25, pos=c(rep(2,length(lambda)-1),4))
text(min(norm.beta), max(det), "Variance vs. Shrinkage", cex=1.5, pos=4)
```

```
# plot trace[Var(b)] vs. length(beta)
with(pdat, {
plot(norm.beta, trace, type="b",
cex.lab=1.25, pch=16, cex=1.5, col=clr, lwd=2,
xlab='shrinkage: ||b|| / max(||b||)',
ylab='variance: trace [Var(b)]')
text(norm.beta, trace, lambda, cex=1.25, pos=c(2, rep(4,length(lambda)-1)))
# text(min(norm.beta), max(det), "Variance vs. Shrinkage", cex=1.5, pos=4)
})
```
prostate *Prostate Cancer Data*

Description

Data to examine the correlation between the level of prostate-specific antigen and a number of clinical measures in men who were about to receive a radical prostatectomy.

Format

A data frame with 97 observations on the following 10 variables.

```
lcavol log cancer volume
lweight log prostate weight
age in years
lbph log of the amount of benign prostatic hyperplasia
svi seminal vesicle invasion
lcp log of capsular penetration
gleason a numeric vector
pgg45 percent of Gleason score 4 or 5
lpsa response
train a logical vector
```
Details

This data set came originally from the (now defunct) ElemStatLearn package.

The last column indicates which 67 observations were used as the "training set" and which 30 as the test set, as described on page 48 in the book.

Note

There was an error in this dataset in earlier versions of the package, as indicated in a footnote on page 3 of the second edition of the book. As of version 2012.04-0 this was corrected.

})

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Source

Stamey, T., Kabalin, J., McNeal, J., Johnstone, I., Freiha, F., Redwine, E. and Yang, N (1989) Prostate specific antigen in the diagnosis and treatment of adenocarcinoma of the prostate II. Radical prostatectomy treated patients, *Journal of Urology*, 16: 1076–1083.

Examples

```
data(prostate)
str( prostate )
cor( prostate[,1:8] )
prostate <- prostate[, -10]
prostate.mod <- lm(lpsa ~ ., data=prostate)
vif(prostate.mod)
py <- prostate[, "lpsa"]
pX <- data.matrix(prostate[, 1:8])
pridge <- ridge(py, pX, df=8:1)
pridge
# univariate ridge trace plots
traceplot(pridge)
traceplot(pridge, X="df")
# bivariate ridge trace plots
plot(pridge)
pairs(pridge)
```
ridge *Ridge Regression Estimates*

Description

The function ridge fits linear models by ridge regression, returning an object of class ridge designed to be used with the plotting methods in this package.

It is also designed to facilitate an alternative representation of the effects of shrinkage in the space of uncorrelated (PCA/SVD) components of the predictors.

The standard formulation of ridge regression is that it regularizes the estimates of coefficients by adding small positive constants λ to the diagonal elements of $X^{\top}X$ in the least squares solution to achieve a more favorable tradeoff between bias and variance (inverse of precision) of the coefficients.

$$
\widehat{\beta}_k^{\text{RR}} = (\mathbf{X}^\top \mathbf{X} + \lambda \mathbf{I})^{-1} \mathbf{X}^\top \mathbf{y}
$$

Ridge regression shrinkage can be parameterized in several ways.

- If a vector of lambda values is supplied, these are used directly in the ridge regression computations.
- Otherwise, if a vector df can be supplied the equivalent values for effective degrees of freedom corresponding to shrinkage, going down from the number of predictors in the model.

In either case, both lambda and df are returned in the ridge object, but the rownames of the coefficients are given in terms of lambda.

coef extracts the estimated coefficients for each value of the shrinkage factor

vcov extracts the estimated $p \times p$ covariance matrices of the coefficients for each value of the shrinkage factor.

best extracts the optimal shrinkage values according to several criteria: HKB: Hoerl et al. (1975); LW: Lawless & Wang (1976); GCV: Golub et al. (1975)

Usage

```
ridge(y, \ldots)## S3 method for class 'formula'
ridge(formula, data, lambda = 0, df, svd = TRUE, contrasts = NULL, \ldots)
## Default S3 method:
ridge(y, X, lambda = 0, df, svd = TRUE, ...)## S3 method for class 'ridge'
coef(object, ...)
## S3 method for class 'ridge'
print(x, digits = max(5, getOption("digits") - 5), ...)
## S3 method for class 'ridge'
vcov(object, ...)
best(object, ...)
## S3 method for class 'ridge'
```
best(object, ...)

Arguments

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Details

If an intercept is present in the model, its coefficient is not penalized. (If you want to penalize an intercept, put in your own constant term and remove the intercept.)

The predictors are centered, but not (yet) scaled in this implementation.

A number of the methods in the package assume that lambda is a vector of shrinkage constants increasing from l ambda[1] = 0, or equivalently, a vector of df decreasing from p.

Value

A list with the following components:

A data.frame with one row for each of the HKB, LW, and GCV criteria

Author(s)

Michael Friendly

References

Hoerl, A. E., Kennard, R. W., and Baldwin, K. F. (1975), "Ridge Regression: Some Simulations," *Communications in Statistics*, 4, 105-123.

Lawless, J.F., and Wang, P. (1976), "A Simulation Study of Ridge and Other Regression Estimators," *Communications in Statistics*, 5, 307-323.

Golub G.H., Heath M., Wahba G. (1979) Generalized cross-validation as a method for choosing a good ridge parameter. *Technometrics*, 21:215–223. [doi:10.2307/1268518](https://doi.org/10.2307/1268518)

See Also

[lm.ridge](#page-0-0) for other implementations of ridge regression

[traceplot](#page-30-1), [plot.ridge](#page-17-1), [pairs.ridge](#page-12-1), [plot3d.ridge](#page-20-1), for 1D, 2D, 3D plotting methods

[pca.ridge](#page-14-1), [biplot.ridge](#page-3-2), [biplot.pcaridge](#page-3-1) for views in PCA/SVD space

[precision.ridge](#page-23-2) for measures of shrinkage and precision

Examples

```
#\donttest{
# Longley data, using number Employed as response
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])
lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)
# same, using formula interface
lridge <- ridge(Employed ~ GNP + Unemployed + Armed.Forces + Population + Year + GNP.deflator,
data=longley, lambda=lambda)
coef(lridge)
# standard trace plot
traceplot(lridge)
# plot vs. equivalent df
traceplot(lridge, X="df")
pairs(lridge, radius=0.5)
#}
data(prostate)
py <- prostate[, "lpsa"]
pX <- data.matrix(prostate[, 1:8])
pridge \le- ridge(py, pX, df=8:1)
pridge
plot(pridge)
pairs(pridge)
```
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```
traceplot(pridge)
traceplot(pridge, X="df")
# Hospital manpower data from Table 3.8 of Myers (1990)
data(Manpower)
str(Manpower)
mmod <- lm(Hours ~ ., data=Manpower)
vif(mmod)
# ridge regression models, specified in terms of equivalent df
mridge <- ridge(Hours ~ ., data=Manpower, df=seq(5, 3.75, -.25))
vif(mridge)
# univariate ridge trace plots
traceplot(mridge)
traceplot(mridge, X="df")
# bivariate ridge trace plots
plot(mridge, radius=0.25, labels=mridge$df)
pairs(mridge, radius=0.25)
# 3D views
# ellipsoids for Load, Xray & BedDays are nearly 2D
plot3d(mridge, radius=0.2, labels=mridge$df)
# variables in model selected by AIC & BIC
plot3d(mridge, variables=c(2,3,5), radius=0.2, labels=mridge$df)
# plots in PCA/SVD space
mpridge <- pca(mridge)
traceplot(mpridge, X="df")
biplot(mpridge, radius=0.25)
```
traceplot *Univariate Ridge Trace Plots*

Description

The traceplot function extends and simplifies the univariate ridge trace plots for ridge regression provided in the plot method for [lm.ridge](#page-0-0)

Usage

```
traceplot(
 x,
 X = c("lambda", "df"),
```

```
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```

```
col = c("black", "red", "darkgreen", "blue", "darkcyan", "magenta", "brown",
  "darkgray"),
pch = c(15:18, 7, 9, 12, 13),
xlab,
ylab = "Coefficient",
xlim,
ylim,
...
```
Arguments

)

Details

For ease of interpretation, the variables are labeled at the side of the plot (left, right) where the coefficient estimates are expected to be most widely spread. If xlim is not specified, the range of the X variable is extended slightly to accommodate the variable names.

Value

None. Used for its side effect of plotting.

Author(s)

Michael Friendly

References

Friendly, M. (2013). The Generalized Ridge Trace Plot: Visualizing Bias *and* Precision. *Journal of Computational and Graphical Statistics*, 22(1), 50-68, doi:10.1080/10618600.2012.681237, <https://www.datavis.ca/papers/genridge-jcgs.pdf>

Hoerl, A. E. and Kennard R. W. (1970). "Ridge Regression: Applications to Nonorthogonal Problems", *Technometrics*, 12(1), 69-82.

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See Also

[ridge](#page-26-1) for details on ridge regression as implemented here

[plot.ridge](#page-17-1), [pairs.ridge](#page-12-1) for other plotting methods

Examples

```
longley.y <- longley[, "Employed"]
longley.X <- data.matrix(longley[, c(2:6,1)])
lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(longley.y, longley.X, lambda=lambda)
traceplot(lridge)
#abline(v=lridge$kLW, lty=3)
#abline(v=lridge$kHKB, lty=3)
#text(lridge$kLW, -3, "LW")
#text(lridge$kHKB, -3, "HKB")
traceplot(lridge, X="df")
```
trans.colors *Make Colors Transparent*

Description

Takes a vector of colors (as color names or rgb hex values) and adds a specified alpha transparency to each.

Usage

trans.colors(col, alpha = 0.5, names = NULL)

Arguments

Details

Colors (col) and alpha need not be of the same length. The shorter one is replicated to make them of the same length.

A vector of color values of the form "#rrggbbaa"

Author(s)

Michael Friendly

See Also

[col2rgb](#page-0-0), [rgb](#page-0-0),

Examples

```
trans.colors(palette(), alpha=0.5)
# alpha can be vectorized
trans.colors(palette(), alpha=seq(0, 1, length=length(palette())))
# lengths need not match: shorter one is repeated as necessary
trans.colors(palette(), alpha=c(.1, .2))
trans.colors(colors()[1:20])
# single color, with various alphas
trans.colors("red", alpha=seq(0,1, length=5))
# assign names
```

```
trans.colors("red", alpha=seq(0,1, length=5), names=paste("red", 1:5, sep=""))
```


vif.ridge *Variance Inflation Factors for Ridge Regression*

Description

The function vif.ridge calculates variance inflation factors for the predictors in a set of ridge regression models indexed by the tuning/shrinkage factor, returning one row for each value of the λ parameter.

Variance inflation factors are calculated using the simplified formulation in Fox & Monette (1992).

The plot.vif.ridge method plots variance inflation factors for a "vif.ridge" object in a similar style to what is provided by [traceplot](#page-30-1). That is, it plots the VIF for each coefficient in the model against either the ridge λ tuning constant or it's equivalent effective degrees of freedom.

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Usage

```
## S3 method for class 'ridge'
vif(mod, ...)
## S3 method for class 'vif.ridge'
print(x, digits = max(4, getOption("digits") - 5), ...)## S3 method for class 'vif.ridge'
plot(
  x,
  X = c("lambda", "df"),Y = c("vif", "sqrt"),
  col = c("black", "red", "darkgreen", "blue", "darkcyan", "magenta", "brown",
    "darkgray"),
  pch = c(15:18, 7, 9, 12, 13),
  xlab,
  ylab,
  xlim,
  ylim,
  ...
\mathcal{L}
```
Arguments

Value

vif returns a "vif.ridge" object, which is a list of four components

vif a data frame of the same size and shape as coef{mod}. The columns correspond to the predictors in the model and the rows correspond to the values of lambda in ridge estimation.

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Author(s)

Michael Friendly

References

Fox, J. and Monette, G. (1992). Generalized collinearity diagnostics. *JASA*, 87, 178-183, [doi:10.1080](https://doi.org/10.1080/01621459.1992.10475190)/ [01621459.1992.10475190.](https://doi.org/10.1080/01621459.1992.10475190)

See Also

[vif](#page-0-0), [precision](#page-23-1)

Examples

```
data(longley)
lmod <- lm(Employed ~ GNP + Unemployed + Armed.Forces + Population +
                      Year + GNP.deflator, data=longley)
vif(lmod)
lambda <- c(0, 0.005, 0.01, 0.02, 0.04, 0.08)
lridge <- ridge(Employed ~ GNP + Unemployed + Armed.Forces +
                          Population + Year + GNP.deflator,
             data=longley, lambda=lambda)
coef(lridge)
# get VIFs for the shrunk estimates
vridge <- vif(lridge)
vridge
names(vridge)
# plot VIFs
pch <- c(15:18, 7, 9)
clr <- c("black", rainbow(5, start=.6, end=.1))
plot(vridge,
     col=clr, pch=pch, cex = 1.2,
     xlim = c(-0.02, 0.08)plot(vridge, X = "df",
     col=clr, pch=pch, cex = 1.2,
     xlim = c(4, 6.5)# Better to plot sqrt(VIF). Plot against degrees of freedom
```
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plot(vridge, X = "df", Y="sqrt", col=clr, pch=pch, cex = 1.2, $xlim = c(4, 6.5)$

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