Package ‘RBE3’

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Type Package

Title Estimation and Additional Tools for Quantile Generalized Beta Regression Model

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Description Provide estimation and data generation tools for the quantile generalized beta regression model. For details, see Bourguignon, Gallardo and Saulo <arXiv:2110.04428> The package also provides tools to perform covariates selection.

Depends R (>= 4.0.0), stats

Imports pracma, gtools

License GPL (>= 2)

NeedsCompilation no

Repository CRAN

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Contents

`BE3` *The Generalized Beta Distribution*

Description

Density, distribution function, quantile function and random generation for the generalized beta distribution.

Usage

```
dBE3(x, mu = 0.5, alpha = 1, beta = 1, tau = 0.5, log = FALSE)
pBE3(q, mu = 0.5, alpha = 1, beta = 1, tau = 0.5, lower.tail = TRUE, log.p = FALSE)
qBE3(p, mu = 0.5, alpha = 1, beta = 1, tau = 0.5)
rBE3(n, mu = 0.5, alpha = 1, beta = 1, tau = 0.5)
```

Arguments

`x, q`	vector of quantiles.
`p`	vector of probabilities.
`n`	number of observations.
`mu`	vector of τ-quantiles of the distribution.
`alpha, beta`	shape parameters of the distribution
`tau`	corresponding quantile of the distribution $(0 < \tau < 1)$
`log, log.p`	logical; if TRUE, probabilities p are given as $\log p$.
`lower.tail`	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$.

Details

The probability density function for the generalized beta distribution is

$$f(y; \lambda, \alpha, \beta) = \frac{\lambda^\alpha y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha, \beta)[1-(1-\lambda)y]^{\alpha+\beta}}, \quad 0 < y < 1,$$

where $\alpha, \beta > 0$ and $\lambda > 0$. We consider the reparameterization in terms of the τ-quantile of the distribution, say $0 < \mu < 1$, taking

$$\lambda = \frac{(1-\mu)}{\mu} \frac{z_{\alpha,\beta}(\tau)}{[1 - z_{\alpha,\beta}(\tau)]},$$

with $z_{\alpha,\beta}(\tau)$ denoting the τ-quantile of the usual beta distribution with shape parameters α and β. The cumulative distribution function is given by

$$F(y; \lambda, \alpha, \beta) = I_{\lambda x/(1+\lambda x - x)}(\alpha, \beta), \quad 0 < y < 1,$$

where $I_x(\alpha,\beta) = B_x(\alpha,\beta)/B(\alpha,\beta)$ is the incomplete beta funcion ratio, $B_x(\alpha,\beta) = \int_0^x w^{\alpha-1}(1-w)^{\beta-1}dw$ is the incomplete beta function and $B(\alpha,\beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha+\beta)$ is the ordinary beta function. The quantile of the distribution can be represented as

$$q(\tau;\lambda,\alpha,\beta) = \frac{z_{\alpha,\beta}(\tau)}{\lambda[1 - z_{\alpha,\beta}(\tau)] + z_{\alpha,\beta}(\tau)}, \quad 0 < \tau < 1.$$

Random generation can be performed using the stochastic representation of the model. If $X_1 \sim \text{Gamma}(\alpha,\theta_1)$ and $X_2 \sim \text{Gamma}(\beta,\theta_2)$, then

$$\frac{X_1}{X_1 + X_2} \sim GB3(\alpha,\beta,\lambda),$$

where $\lambda = \theta_1/\theta_2$.

Value

dBE3 gives the density, pBE3 gives the distribution function, qBE3 gives the quantile function, and rBE3 generates random deviates.

The length of the result is determined by n for rBE3, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than n are recycled to the length of the result. Only the first elements of the logical arguments are used.

Author(s)

Diego Gallardo and Marcelo Bourguignon

References

Libby, D. L. and Novick, M. R. (1982). Multivariate generalized beta-distributions with applications to utility assessment. Journal of Educational Statistics, 7.

Examples

```
rBE3(20, mu=0.5, alpha=2, beta=1)
dBE3(c(0.4,0.7), mu=0.5, alpha=2, beta=1)
pBE3(c(0.4,0.7), mu=0.5, alpha=2, beta=1)
```

`BE3.backward` *backward stepwise regression for RBE3 model based on the AIC criterion or significance.*

Description

BE3.backward.crit implements the covariates selection based on backward and the Akaike's information criteria (AIC). BE3.backward.sign implements the covariates selection based on backward and significance of the covariates.

Usage

```
BE3.backward.crit(data, tau = 0.5, link.mu = "logit")
```

Arguments

data	a list containing the response vector (y), and the matrices to model μ, the τ-quantile of distribution, and the shape parameters α and β, labeled as Z_1, Z_2 and Z_3, respectively.
tau	the quantile of the distribution to be modelled ($0 < \tau < 1$).
link.mu	link function to be used for μ: logit (default), probit, loglog or cloglog.

Value

A list containg the covariates to be included for modelling μ, α and β, respectively.

Author(s)

Diego Gallardo and Marcelo Bourguignon.

Examples

```
##Simulating two covariates
set.seed(2100)
x1<-rnorm(200); x2<-rbinom(200, size=1, prob=0.5)
##Desing matrices: Z1 includes x1 and x2,
##Z2 includes only x1 and Z3 includes only x2
Z1=model.matrix(~x1+x2);Z2=model.matrix(~x1);Z3=model.matrix(~x2)
##Fixing parameters
theta=c(1, 0.2, -0.5); nu=c(0.5,-0.2); eta=c(-0.5, 0.3); tau=0.4
mu=plogis(Z1%*%theta); alpha=exp(Z2%*%nu); beta=exp(Z3%*%eta)
y=rBE3(200, mu, alpha, beta, tau=tau)
data=list(y=y, Z1=Z1, Z2=Z2, Z3=Z3)
BE3.backward.crit(data, tau = tau)
```

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gumbel2 *The Gumbel2 distribution*

Description

Density, distribution function and quantile function for the Gumbel2 distribution.

Usage

```
dgumbel2(x, log=FALSE)
pgumbel2(q)
qgumbel2(p)
```

Arguments

`x, q`	Vector of quantiles.
`p`	Vector of probabilities.
`log`	logical; if TRUE, probabilities p are given as log(p).

Details

The cumulative distribution function for the Gumbel2 distribution is given by $F(x) = 1 - exp(-exp(x))$.

Value

dgumbel2 gives the density, pgumbel2 gives the distribution function and qgumbel2 gives the quantile function.

The length of the result is determined by the maximum of the lengths of the numerical arguments.

Author(s)

Diego Gallardo and Marcelo Bourguignon.

Examples

```
dgumbel2(c(4,10))
pgumbel2(c(4,10))
qgumbel2(c(0.1, 0.5))
```

`ML.BE3` *Perform the parameter estimation for the Generalized beta distribution*

Description

ML.BE3 computes the maximum likelihood estimates based on the maximum likelihood method.

Usage

```
ML.BE3(data, tau = 0.5, link.mu = "logit")
```

Arguments

`data`	a list containing the response vector (y), and the matrices to model μ, the τ-quantile of distribution, and the shape parameters α and β, labeled as Z_1, Z_2 and Z_3, respectively.
`tau`	the quantile of the distribution to be modelled $(0 < \tau < 1)$.
`link.mu`	link function to be used for μ: logit (default), probit, loglog or cloglog.

Details

Covariates are included as $g_1(\mu_i(\tau)) = \mathbf{Z}_{1i}^\top \boldsymbol{\theta}(\tau)$, $g_2(\alpha_i(\tau)) = \mathbf{Z}_{2i}^\top \boldsymbol{\nu}(\tau)$ and $g_3(\beta_i(\tau)) = \mathbf{Z}_{3i}^\top \boldsymbol{\eta}(\tau)$, where $\boldsymbol{\theta}(\tau) = (\theta_1(\tau), \ldots, \theta_{r_1}(\tau))$, $\boldsymbol{\nu}(\tau) = (\nu_1(\tau), \ldots, \nu_{r_2}(\tau))$ and $\boldsymbol{\eta}(\tau) = (\eta_1(\tau), \ldots, \eta_{r_3}(\tau))$, where r_1, r_2 and r_3 are the dimensions of Z_1, Z_2 and Z_3, respectively. Initial values for $\boldsymbol{\theta}(\tau)$ are used as the coefficients for the linear regresion in logit(y_i) using the elements of $\mathbf{Z}_{1i}^\top$ as regressors. Initial values for the other coefficients are considered as zeros.

Value

a list containing the following elements

`estimate`	A matrix with the estimates
`logLik`	The maximum likelihood values attached by the estimates parameters

Author(s)

Diego Gallardo and Marcelo Bourguignon.

References

Bourguignon, M., Gallardo, D.I., Saulo, H. (2023) A parametric quantile beta regression for modeling case fatality rates of COVID-19. Submitted.

Examples

```
##Simulating two covariates
set.seed(2100)
x1<-rnorm(200); x2<-rbinom(200, size=1, prob=0.5)
##Desing matrices: Z1 includes x1 and x2,
##Z2 includes only x1 and Z3 includes only x2
Z1=model.matrix(~x1+x2);Z2=model.matrix(~x1);Z3=model.matrix(~x2)
##Fixing parameters
theta=c(1, 0.2, -0.5); nu=c(0.5,-0.2); eta=c(-0.5, 0.3); tau=0.4
mu=plogis(Z1%*%theta); alpha=exp(Z2%*%nu); beta=exp(Z3%*%eta)
y=rBE3(200, mu, alpha, beta, tau=tau)
data=list(y=y, Z1=Z1, Z2=Z2, Z3=Z3)
ML.BE3(data, tau=tau)
```

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